Wavelet-based multifractal analysis of global environmental Oscillations

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Abstract

We analyzed the multifractal behavior of the monthly El Nino/Southern Oscillation index (Nino3.4), the monthly Indian Ocean Dipole Mode Index (DMI) and the monthly North Atlantic Oscillation index (NAO) by using the wavelet transform. We interpreted the climate change by a new viewpoint of fractal. When the Nino3.4 and DMI had high coherency, the strong multifractality was observed. Change from multifractal to monofractal was observed at the 1976/77 regime shift for Nino3.4 and DMI and was observed at the 1988/89 regime shift for NAO. We found that a regime shift occured when the fluctuation was large and the multifractality was strong.

1. Introduction

Relationships between the multifractality and the development of an El Nino/Southern Oscillation, Indian Ocean Dipole Mode and North Atlantic Oscillation have not been documented so far. The wavelet transform allows a reliable multifractal analysis to be performed (Muzy et al. 1991). The properties of the wavelet transform make wavelet methods attractive for the analysis of complex non-stationary time series. In climate field the following studies concering fractal are performed. The fractal dimension H was explored to measure the noise characteristics of Nino3.4 and the results show that the oscillation pattern of H corresponds mostly to development of El Nino (Huang and Morimoto 2008). The multifractal properties of daily rainfall were investigated in two contrasting climates: as east Asian monsoon climate with an extreme rainfall variability and a temperate climate with a moderate rainfall variability (Svensson et al. 1996).

As the phase transition as well as change of state occurs accompanying with the change of fractality, we will explain the change of climate by examing the fractality. The relationship between the multifractality and the change of climate is important. We interpret the climate change by a new viewpoint of fractal. In this study, we examine changes of multifractality in Nino3.4, DMI and NAO by using the wavelet transform and we compare them with changes of climate.

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2. Data and method of analysis

In this study, we used the monthly Nino3.4 from January 1950 through December 2008, DMI from January 1960 through December 2008 and NAO from January 1950 through December 2008 as explained below. The El Nino/Southern Oscillation index (Nino3.4), which is used as a measure of the amplitude of an ENSO event, is defined as the monthly sea surface temperature averaged over the east central tropical Pacific (5°N-5°S, 120°W-170°W) and is provided by NOAA Climate Prediction Center in USA (CPC). The Indian Ocean Dipole Mode Index (DMI), which is the intensity of the Indian Ocean Dipole, is defined as the sea surface temperature anomaly difference between the tropical western Indian Ocean (50°E-70°E, 10°S-10°N) and the tropical southeastern Indian Ocean (90°E-110°E, 10°S-equator). DMI is derived from the Hadley Centre Sea Ice and SST Data Set (HadISST) provided by the Hadley Centre, Met Office, UK. The North Atlantic Oscillation index (NAO) is defined by the sea level pressure (SLP) difference between the Icelandic Low (IL) and the Azores High (AH). NAO used here is provided by CPC. The noise of high frequency whose period is shorter than one year was cut.

We used derivatives of the Daubechies function as the analyzing wavelet. We then estimated the scaling of the partition function $Z_q(a)$, which is defined as the sum of the qth powers of the modulus of the wavelet transform coefficients at scale a. A time window of the SST time series was selected with a period of 72 months, so as to involve ENSO cycle 2~6 years, starting from January. For small scales, we expect

$$Z_q(a) \sim a^{\tau(q)}. \qquad (1)$$

First, we investigate the changes of $Z_q(a)$ on Nino3.4 SST time series at the different scales for each q. The plot of the logarithm of $Z_q(a)$ against the logarithm of time scale is performed. Here $\tau(q)$ is the slope of the linear fitted line of a log-log plot for each q. Next, we plot $\tau(q)$ vs q. The time window was then shifted forward one year and we repeated above. Monofractal signals display a linear $\tau(q)$ spectrum.

For multifractal signals, τ(q) is a nonlinear function. For positive q, $Z_q(a)$ reflects the scaling of the large fluctuations and strong singularities, whereas for negative q, $Z_q(a)$ reflects the scaling of the small fluctuations and weak singularities (Takayasu 1997). Thus, the scaling exponents τ(q) can reveal different aspects of climatic dynamics.

3. Results and Discussion

We show the time series of monthly NINO3.4 for 1950-2008, DMI for 1960-2008 and NAO for 1950-2008 in Figs. 4, 5 and 6, respectively. In NINO3.4 the strong El Nino is observed in 1972, 1983, 1987 and 1997. The strong La Nina is observed in the1970s and 1989. In the 1980s the periodicity of Nino3.4 was strong. In the 1980s and 1990s the periodicity of DMI was strong. In NAO the tendency of negative phase was clear in the 1960s. NAO had a sharp reversal in the 1980s and positive phase in 1988-95 year.

We show the wavelet power spectra of NINO3.4, DMI and NAO using the Morlet wavelet in Fig. 1. When the analysis is focused on amplitude and phase changes, a complex wavelet, like the Morlet wavelet, is the most appropriate. Nino3.4 showed clear period of about 4 years cycles through the 1980s. DMI showed clear period of about 4 years cycles through the 1990s.

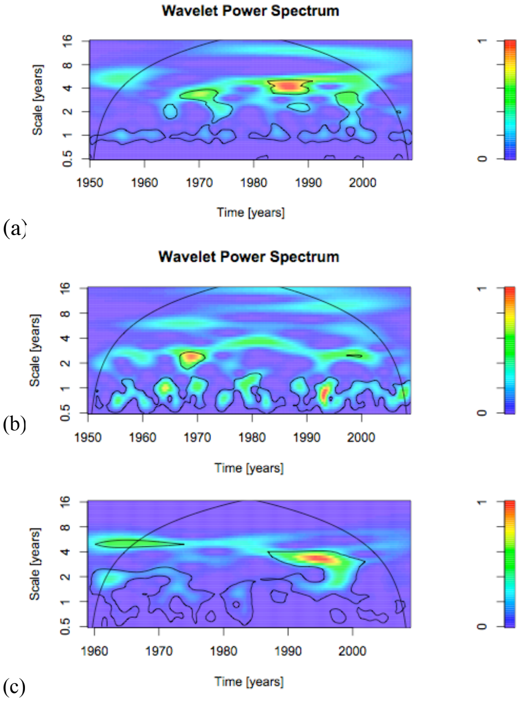
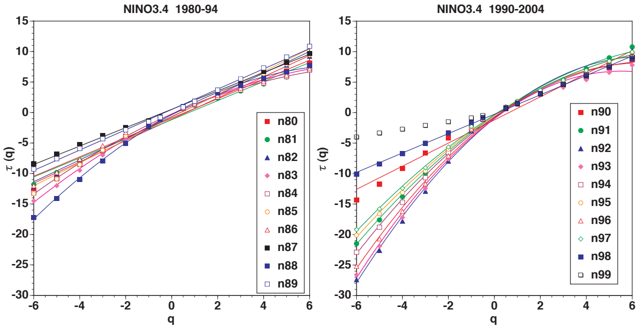


Fig. 1: Wavelet power spectra of NINO3.4 (a), DMI(b) and NAO(c).

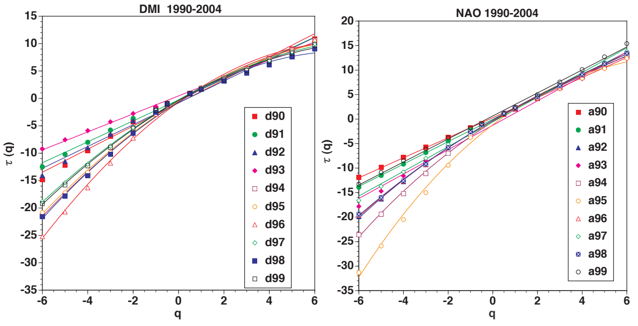
We calculated the multifractal spectrum τ(q) of different moments q for individual records between 1950 and 2004 year of NINO3.4. In Fig. 2(a) the multifractal spectrum τ(q) between 1980 and 1994 year is shown. The Multifractal spectrum τ(q) of n80 was calculated between 1980 and 85 year. That of n81 was calculated between 1981 and 86 year. To examine the change of fractality, the time window was then shifted forward one year and we calculated the multifractal spectrum τ(q) from n80 up to n89. The multifractal spectrum τ(q) between 1990 and 2004 year is shown in Fig. 2(b). A monofractal signal would correspond to a straight line for τ(q) and for a multifractal signal τ(q) is nonlinear. Most of the multifractality observed is due to the negative moments, i.e., the small fluctuations are more inhomogeneous than the big fluctuations. In Fig. 2(a), the data sets were monofractal in the case of n80~82, n85~87 and n89 and were multifractal in the case of n83, n84 and n88. In Fig. 2(b), the data sets were monofractal in the case of n90 and n98 and were multifractal in the case of n91~96 and n97.



(a) (b)

Fig. 2: Multifractal spectrum τ(q) for individual records between 1980 and 1994 year (a) and between 1990 and 2004 year (b) of NINO3.4.

We calculated the multifractal spectrum τ(q) for individual records between 1960 and 2004 year of DMI. The multifractal spectrum τ(q) between 1990 and 2004 year is shown in Fig. 3(a) and the data sets were monofractal in the case of d90~92 and d93 and were multifractal in the case of d94~99. We calculated the multifractal spectrum τ(q) for individual records between 1950 and 2004 year of NAO. The multifractal spectrum τ(q) between 1990 and 2004 year is shown in Fig. 3(b) and the data sets were monofractal in the case of a90, a91, a93, a97 and a99 and were multifractal in the case of a92, a94~96 and a98.



(a) (b)

Fig. 3: Multifractal spectrum τ(q) for individual records between 1990 and 2004 year of DMI (a) and NAO (b).

We show the time series of τ(q=-6) of NINO3.4, DMI and NAO in Figs. 4, 5 and 6, respectively. The value of τ(q=-6) is the index of the fractality and negative large value shows large multifractality. The red square shows monofractality and the green circle shows

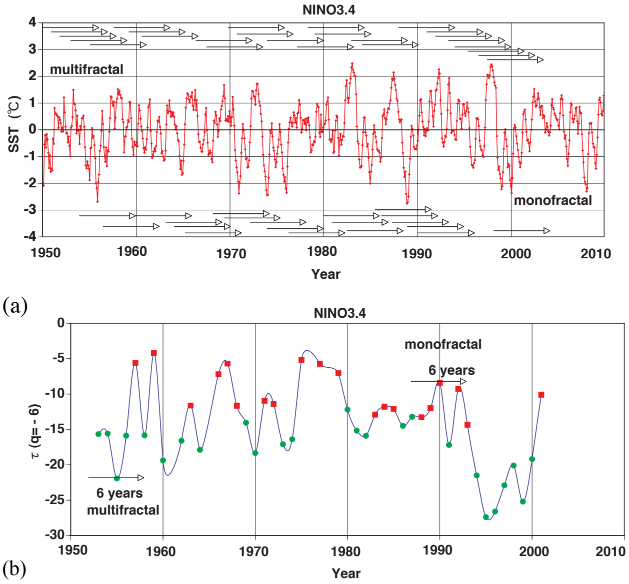


Fig. 4: Time series of NINO3.4(a) which involves the period of monofractal and multifractal. Time series of τ(q=-6) of NINO3.4(b). The red square shows monofractality and the green circle shows multifractality for 6 years centering the year.

multifractality for 6 years centering the year plotted. For example, the green circle in 1953 shows multifractality between 1950 and 1955 in

Fig. 4(b). Time series of NINO3.4, DMI and NAO, which involve the period of monofractal and multifractal are also shown in Figs. 4, 5 and 6, respectively.

In the 1980s the periodicity of Nino3.4 was strong and monofractality is observed. In the 1990s the periodicity of DMI was strong and monofractality was observed. In the 2000s the periodicity of NAO was strong and monofractality was observed. Monofractal signals are homogeneous and have linear properties, while multifractal signals are inhomogeneous and have nonlinear properties (Stanley et al. 1999).

Nino 3.4 and DMI had significant and large jump at the 1976/77 regime shift and changes from multifractal to monofractal were observed as shown in Figs. 4 and 5, respectively. After the 1976/77 regime shift the periodicity of Nino 3.4 and DMI was observed. A regime shift is characterized by an abrupt transition from one quasi-steady climatic state to another, and its transition period is much shorter than the lengths of the individual epochs of each climatic state. The duration between each regime shift is about 10 years. It is becoming increasingly clear that many complex systems have critical thresholds at which the system shifts abruptly from one state to another. In the Earth system, abrupt shift in ocean circulation or climate may occur (Scheffer et al. 2009).

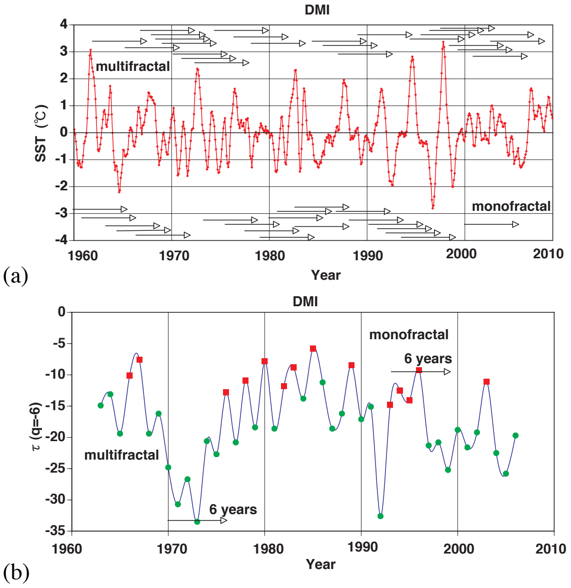


Fig. 5: Same as Fig. 4 for time series (a) and τ(q=-6) of DMI (b).

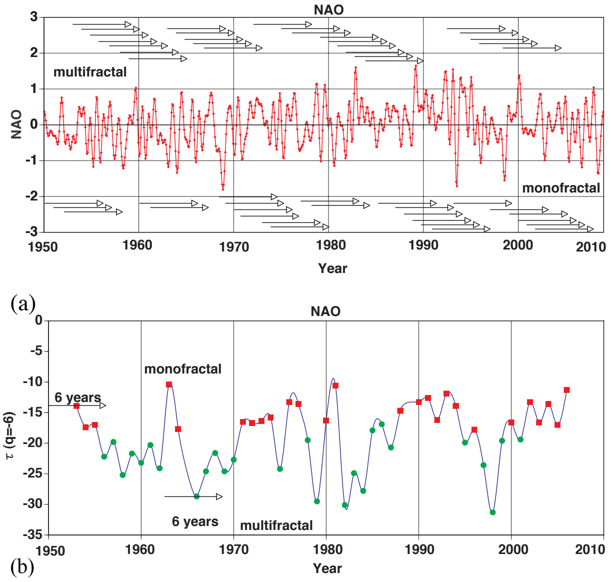


Fig. 6: Same as Fig. 4 for time series (a) and τ(q=-6) of NAO (b).

The 1976/77 regime shift is the most famous one and after that the tropical Pacific was warming, the central North Pacific was cooling, and at the same time, the Aleutian Low (AL) was strengthened. These conditions lasted to the late 1980s. For the regime shift in 1988/89, this tendency, the large jump, was not observed. Although most of the regime shifts (the 1925/26, 1945/46, 1957/58, 1970/71 and 1976/77 shifts) included the tropical variations, the 1988/89 shift was independent of them (Yasunaka and Hanawa 2003). On the contrary, NAO had a significant and large jump at the 1988/89 regime shift and change from multifractal to monofractal was observed as shown in Fig. 6. After the 1988/89 regime shift the periodicity of NAO was observed and the global temperature became warm. For the 1976/77 regime shift this tendency, the large jump, was not observed.

The regime shifts can be divided into two groups: (1) The 1976/77 regime shift is closely linked with the tropical Pacific and the Indian Ocean variations. (2) The 1988/89 regime shift is independent of these tropical variations and depends on the another dominant mode of variation corresponding to the Arctic Oscillation (Yasunaka and Hanawa 2003). These results agreed with our results that change from multifractal to monofractal was observed at the 1976/77 regime shift for Nino3.4 and DMI and was observed at the 1988/89 regime shift for NAO, which tendency is similar to that of Arctic Oscillation.

We show the wavelet coherency and phase using the Morlet wavelet between Nino3.4 and DMI in Figs.7(middle) and 7(bottom), respectively. Time series of τ(q=-6) of NINO3.4 and DMI are also shown in Fig.7(top). Nino3.4 and DMI showed high coherency of 1-2 and 2-4 years signals through 1968-72 and 1995-2002 year, respectively and the difference of the phase are NINO3.4 being late and 0, respectively. In particular, the coherency for 1995-2002 year was very strong. When Nino3.4 and DMI had high coherency, the strong multifractality was observed for 1968-74 and 1994-2001 year in Nino3.4 and DMI. Hence, we may conclude as follows. When Nino3.4 and DMI had high coherency, with increasing fluctuation by interaction, the strong multifractality was observed. And in the case of high coherency the dipole mode events coincided with strong ENSO events in 1972 and 1997.

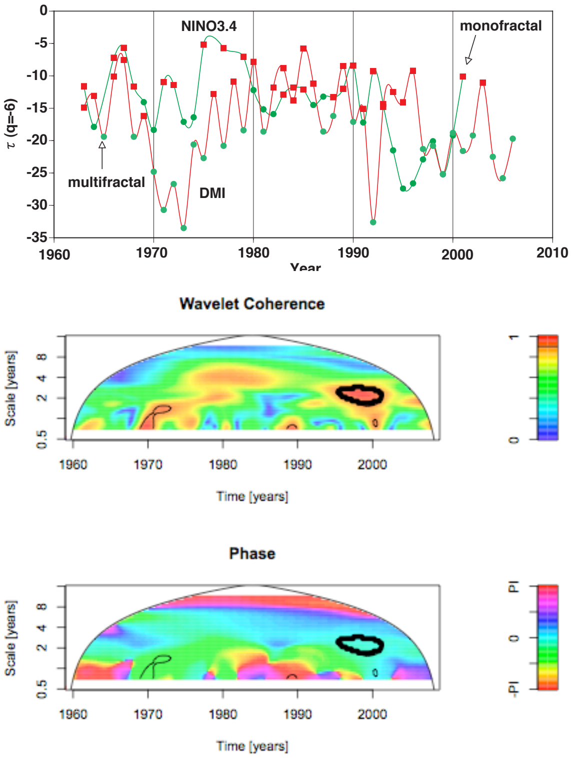


Fig.7: Time series of τ(q=-6) of NINO3.4 index and DMI (top). Wavelet coherency (middle) and phase (bottom) between Nino3.4 index and DMI. The thick black contour encloses regions of greater than 95% confidence.

Before the 1976/77 regime shift, the Nino3.4 and DMI had high coherency, and the multifractality was observed. Change from multifractal to monofractal was observed at the 1976/77 regime shift for Nino3.4 and DMI and was observed at the 1988/89 regime shift for NAO. Hence we infer that a regime shift occured when the fluctuation was large and the multifractality was strong. When the strong multifractality was observed, the climatic state changed and the regime shift occured.

Change from multifractal to monofractal is observed in human heart rate. The healthy human heart rate is a typical example of signals showing long-range temporal correlations (Peng et al. 1993) and multifractal scaling properties (Ivanov et al. 1999). The origin of the complex dynamics of heart rate has been attributed to antagonistic activity of the two branches of the autonomic nervous system: the parasympathetic and the sympathetic nervous systems, respectively, decreasing and increasing heart rate (Peng et al. 1993). In patients with congestive heart failure, the long-range correlated fluctuations decrease and the multifractality of heart rate dynamics is lost and the monofractal character is shown (Ivanov et al. 1999). The control mechanisms regulating the heart rate might interact as part of a coupled cascade of feedback loops in a system operating far from equilibrium. The similar mechanism may control the change from multifractal to monofractal observed at the regime shifts for Nino3.4, DMI and NAO.

4. Conclusion

We analyzed the multifractal behavior of the monthly El Nino/Southern Oscillation index (Nino3.4), the monthly Indian Ocean Dipole Mode Index (DMI) and the monthly North Atlatic Oscillation index (NAO) by using the wavelet transform. Main results were obtained as follows: (1) When the Nino3.4 and DMI had high coherency, with increasing fluctuation by interaction, the strong multifractality was observed. (2) Change from multifractal to monofractal was observed at the 1976/77 regime shift for Nino3.4 and DMI and was observed at the 1988/89 regime shift for NAO. After the regime shift the periodicity of Nino 3.4, DMI and NAO was observed. (3) We found that a regime shift occured when the fluctuation was large and the multifractality was strong.

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